# Reason and Intuition[*]

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In this paper I will approach the subject of intuition from a different angle from what has been usual in the philosophy of mathematics, by beginning with some descriptive remarks about Reason and observing that something that has been called intuition arises naturally in that context. These considerations are quite general, not specific to mathematics. The conception of intuition might be called that of rational intuition; indeed the conception is a much more modest version of conceptions of intuition held by rationalist philosophers. Moreover, it answers to a quite widespread use of the word "intuition" in philosophy and elsewhere. But it does not obviously satisfy conditions associated with other conceptions of intuition that have been applied to mathematics. Intuition in a sense like this has, in writing about mathematics, repeatedly been run together with intuition in other senses. In the last part of the paper a little will be said about the connections that give rise to this phenomenon.

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[*] An abridgement of an earlier version of this paper was presented to a session on Mathematical Intuition at the 20th World Congress of Philosophy in Boston, August 11, 1998. Some parts of the paper, in particular §1 and the discussion of arithmetic in §2, derive from a paper that has been presented to several audiences. It is hard to determine what comments from those occasions have been helpful specifically for the present paper, but David Brink, Patricia and Philip Kitcher, Gila Sher, William Tait, and Steven Yalowitz deserve mention. I wish to thank Mark Steiner and members of the audience at the Congress session for comments on the present paper.

§1. *Reason*. Very generally, I will say that reason is in play when actions, including assertions and related speech acts, are supported or defended by reasons. But the same characterization clearly generalizes further to beliefs and other thoughts that have something like assertive force. However, the giving of a reason might count as an exercise of reason only if it is a good reason or at least has some degree of support by further reasons. Several features of Reason should be mentioned:

a. Reasons are themselves supported by reasons. This arises naturally from the fact that in giving reasons one seeks the agreement or support of others, since reasons for one's reasons can be given in response to their being questioned. The fact that reasons are themselves supported by reasons links reason with *argument*, since iteration of such support gives rise to chains of reasoning. The natural way in which such chains arise is furthermore an entering wedge for logic, although logic is not mentioned in the characterization, and the discussion of reasons in the theory of action and in moral philosophy does not put logic at center stage.

The questioning of steps in an argument and the giving of further reasons for them has the result that some premisses and inferences become explicit that were not before. The buck has to stop somewhere (for the immediate argumentative purpose, if not in any final way), and one place where it characteristically stops is with simple, elementary logical inferences. In modern mathematics, the axiomatic method makes it possible to give an idealized but quite workable and highly illuminating model of proof, in which proofs are represented as deductions from well-defined axioms with the help of some definite logic, in central cases first-order quantificational logic with identity.[1] Then one has identified a central factor in mathematical knowledge that would on almost any account be attributed to reason, namely logic.

If we ask why the buck should stop with elementary logical inferences, one part of the answer is that they are obvious. Though this is not uncontested even for all of first-order logic, as the debates surrounding constructivism showed, I shall put obviousness aside for the moment. It is noteworthy that logic combines a high degree of obviousness with a maximal degree of generality. A characteristic way of justifying a statement or action by reasons is to put it under general rules or principles. This leads us to search for general reasons, and to regard generality as a virtue in reasons.[2] Thus their generality is another reason for treating elementary logical inferences as basic. The same logic will be common to reasoning about a wide variety of subject matters, or reasoning from different assumptions about the same subject matter. This is of course important because it makes possible very general results about mathematical theories and about reasoning in other domains of knowledge. Because of their high degree of obviousness and apparently maximal generality, we do not seem to be able to give a justification of the most elementary logical principles that is not to some degree circular, in that inferences codified by logic will be used in the

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[1] I ignore the fact that it is for some purposes more appropriate to replace certain mathematical axioms (such as mathematical induction) by rules, which are of course not logical if the axioms they replace are not.

[2] It is not so in every context, as I was reminded by Ernesto Napoli.

justification.[3]

b. This brings me to what I take to be one of the central features of reason, another reflection of the fact that in the giving of reasons the buck has to stop somewhere. A wide variety of statements and inferences, not just simple logical ones, are accepted without carrying the argument further. They strike those who make them as true or as evident, prior to any sense of how to construct an argument for them, at least not one that proceeds from premisses that are *more* evident by inferences that are also more evident. This is not to say that there is no argument for the statement possible at all, but the possible arguments are of a more indirect character or have premisses that are no more evident.[4] They might be perceptual judgments, where a relation to an external event, namely a perception, is what prompts their acceptance. But if there is no such relation, we will say that the statement is treated as "intrinsically plausible."[5] It is a feature of reason that some statements, even those that have the character of principles, are treated in this way, and there is no alternative to so treating some statements or inferences. In mathematics, in addition to logical inferences these are typically axioms.

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[3] Of course this does not mean that nothing useful can be said in justification of logical principles.

[4] For some statements that we might accept as having a high degree of intrinsic plausibility, we don't rely on it because they can just as well be derived from others, which are preferred as premisses for various reasons.

[5] I have believed that I derived this phrase from John Rawls, but I have not located it in his writings.

I want to emphasize that the fact that we do rely on the intrinsic plausibility of principles, even where it is unexplained and not something all would agree on, is a fact about *reason*, in the sense of being a fact about what counts as a reason for what. This is certainly true in practice, and I would argue that there is no way around it. There may be vanishingly few principles that we cannot avoid assuming, but if we are not to be skeptics (in the ancient sense, suspending judgment about everything), we have to accept some. This fact comes to be a fact about reason, of course, only because inquiries (and probably also practical decisions) where principles are involved exhibit this feature when they satisfy other criteria of rationality or reasonableness.

I do not use the term "obviousness" or "self-evidence" to describe what I have in mind here. Both imply a higher degree of evidence than I am aiming at; the conception itself is intended to be neutral as to how high a degree of evidence intrinsic plausibility entails. What is indispensable is enough credibility to allow inquiry to proceed and to give the more holistic considerations mentioned under the following headings (c)-(e) something to work with.[6] "Obvious" also covers too wide a range of cases: a statement may be obvious more or less intrinsically, or perceptually, or because of the ready availability of a convincing argument for it. I want to use the term "self-evident" in a rather specific sense. Roughly, a statement is self-evident if it is seen to be true by anyone who has sufficiently clearly in mind what it means, or who exercises a sufficiently clear understanding of it. If one does have such an understanding, it is then only by

[6] In this respect the conception is akin to the conception of initial credibility in Goodman 1952. I owe this observation to Sidney Morgenbesser.

exercising a kind of detachment that one can doubt it, if at all.[7] The notion of self-evidence is often regarded as not a very useful notion, perhaps as having no application. And one can see why: As I have formulated it, it raises the question when an understanding is sufficiently clear, and moreover in actual cases, such as that of the law of excluded middle, there are disputes about what meaning of the statement is *available*. But it is useful to keep the notion of self-evidence in view at least for purposes of comparison.

A natural question to ask is what limits there are on the statements that can be treated as intrinsically plausible. As I have explained matters, in mathematics they will be in the first instance axioms, since if a part of mathematics is formulated axiomatically other statements will be derived. We need to consider inferences as well as statements, and basic logical inferences will also be treated in this way; indeed, such inferences are very frequently regarded as self-evident, and in some cases that is reasonable, for example conjunction-introduction and elimination. I will not, however, be concerned with the question whether in general basic logical inferences are self-evident. But it does seem unavoidable to treat some logic as intrinsically plausible, since at least a minimum of logic will be presupposed in any other form of verification of knowledge with significant systematic structure.

As I have said perceptual judgments should not be regarded as intrinsically plausible, although they share the character of being accepted in the absence of argument. The reason is that their plausible or evident character depends on their being made in the context of an event outside the judgment,

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[7] One can presumably apply the same criterion to inferences and their validity.

namely the perception, and so is dubiously called intrinsic. The question might then arise whether statements that are intuitively known on the basis of the Hilbertian intuition discussed in other writings of mine might be intrinsically plausible or evident.[8] That seems to me a borderline case, and the same might be true in other cases where imaginative thought-experiments are involved.

c. We have set out two features of reason that seem to me fundamental, the importance of argument, which gives rise to the central role of logic, and the fact that some statements that play the role of principles are regarded as plausible (and possibly even evident) without themselves being the conclusions of arguments, or at least not on the ground of the availability of such arguments. A feature of the structure of knowledge that can be viewed as a third feature of reason is the search for systematization. Logic organizes arguments so that they are application of general rules of principles, and then these principles serve to unify the case at hand with others. In mathematics, systematization is manifested in a very particular way, through the axiomatic method.

d. Systematization makes possible a fourth feature, what I will call the dialectical relation of principles, that is high-level generalizations, generally of a theoretical character, and lower-level statements or judgments that we accept, possibly on the basis of perception but also of a non-perceptual character, such as moral judgments.[9] Statements of either level may be plausible more or less intrinsically apart from their connections with statements of the other. A suitable systematization may derive the lower-level judgments from the principles, and

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[8] In particular Parsons 1980, 1993, 1998.

[9] These levels need not be the highest and the lowest. But for simplicity I consider only two levels at a time.

this can serve to reinforce both. It may derive them with some correction, so that the original lower-level judgments come to be seen as not exactly true, but then what is wrong with them should be explained. It may be that some sets of principles lead to systematizations that square much better with the lower-level judgments than others, and this is a ground for choosing those that square better.

Forty years ago Nelson Goodman suggested that one should view the evident character of logic in this dialectical way.[10] There are particular inferences and arguments that we accept. Rules of deductive inference are invalid if they conflict with these judgments of the validity of particular inferences. Particular inferences are valid if they conform to the rules, and not if they do not. Goodman says that the circularity involved is "virtuous". He indicates clearly that there can be a process of mutual correction, which then has a back-and-forth character, so that harmony of accepted principles and accepted particular inferences is a kind of equilibrium. Although the picture is attractive, it is not so easy to identify the process at work in the actual history of logic, although one possible reason for this could be that with respect to elementary inferences, the decisive steps took place too far back to be documentable.

Goodman's picture of the justification of logic is at least incomplete, as one can see from the fact that harmony between principles and accepted inferences could obtain for both the classical mathematician and the intuitionist and so could not yield a criterion for choosing between them. In fact, further argument was brought into play from the beginning, generally either directly philosophical or methodological or turning on questions of mathematical fruitfulness and application. That harmony obtains for either intuitionistic or classical logic when

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[10] Goodman 1955, ch. III, §2.

all these considerations are brought in would be disputed; it has certainly not brought about complete agreement.

John Rawls's view of the justification of moral and political theories also gives a place to dialectical interplay between particular judgments and general principles. In fact, it is his discussion of his methodology in *A Theory of Justice* that brought wide attention to this phenomenon. However, to enter into a discussion of Rawls's conception of "reflective equilibrium" would take us too far afield.[11] To avoid possible misunderstanding, however, we should recall that in *Political Liberalism* Rawls emphasizes that reflective equilibrium involves not just particular judgments and highly general principles but also judgments and principles of all levels of generality.[12] The mature conception of reflective equilibrium is more complex than what Goodman described.

A specific incompleteness in Goodman's discussion is that it is not sufficient to regard the relation that we aim at of principles and lower level judgments as simply a matter of the former implying and only implying judgments or inferences that we are disposed to accept. Even in domains other than natural science, in mathematics in particular, principles can possess something like explanatory power, and that can be a reason for accepting them. In one of his tantalizingly brief discussions of justification of mathematical axioms by their consequences, Gödel mentions "shedding so much light on a

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[11] Rawls acknowledges Goodman's discussion as a model (1971, p. 20 n.). The difficulty I have found with Goodman's discussion might be addressed by the distinction between narrow and wide reflective equilibrium; see Rawls 1975, p. 289.

[12] Rawls 1993, pp. 8, 28.

whole field" as one of some properties of axioms such that an axiom having all of them "would have to be accepted in at least the same sense as any well-established physical theory."[13]

e. A fifth and final feature is that Reason, as embodied in what we accept as a reason for what, is that it is an ultimate court of appeal. The reason why "self-evidence" is never the whole story about the evident character of a principle is the same as the reason why judgments based on perception are in principle fallible. In the end we have to decide, on the basis of the whole of our knowledge and the mutual connections of its parts, whether to credit a given instance of self-evidence or a given case of what appears to be perception. There is no perfectly general reason why judgments of either kind cannot be overridden. When Descartes, after entertaining some serious skeptical doubts, was confronted with his inability to doubt "2 + 3 = 5" when he had its content clearly in mind, he then arrived at the idea of a malicious demon who might deceive him even about the matters that he perceived most clearly. We might in the end say that this is not a serious possibility, but when we do come to that conclusion it will be a case of our buttressing the evidence of "2 + 3 = 5" by argument, not in the sense of producing a stronger proof with that conclusion but in the sense of removing a reason not to credit the reason we already have for

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[13] Gödel 1964, p. 261. My attention was called to this remark by Martin 1998, p. 227. Martin argues that determinacy hypotheses in descriptive set theory satisfy the criteria that Gödel gives. His paper is an exemplary discussion of the issues about evidence in mathematics that arise from the fact that some problems in set theory are settled by axioms that are very far from self-evident or even adequately motivated by the iterative conception of set.

accepting it. Something that we ask of arguments of a nice premisses-conclusion structure is that they should be robust under certain kinds of reflection. The same is true of perceptual judgments or purportedly self-evident ones.

§2. *Intuition*. Statements that are taken to be intrinsically plausible are often called intuitions; in fact, something like this is nowadays the most widespread use of the term in philosophy and is by no means confined to philosophy. But so far, although the conception of intuition yielded can in principle apply to a very wide range of propositions, it is in other respects a very weak one. First, it leaves largely open what epistemological weight is to be given to intuitions. Already that would suggest that some tightening of the conception is necessary. Philosophers who have given great epistemic weight to intuition, such as Descartes, Husserl, and Gödel, have held that one only has intuition if one exercises considerable care to eliminate sources of illusion and error. Connected with this is a second consideration: If we think of intuition as a fundamental source of knowledge, then in theoretical matters intuitions should be stable and intersubjective, but in many inquiries what is regarded as intrinsically plausible may depend on that particular context of inquiry, and moreover disagreements in "intuitions" are very common in most fields. Third, no connection has been made between this notion of intuition and that of intuition of *objects*, so prominent in the philosophy of Kant and in writings influenced by him.[14]

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[14] Among such writings should be included much that has been written on the philosophy of mathematics in the nineteenth and twentieth centuries; even writers hostile to Kant have used the term "intuition" in a way that owes at least distant inspiration from him.

I don't think it fruitful to pursue in general the question what epistemic weight "intuition" in this so far very weak sense should have. For one thing, the answer is too obvious that it would depend on the subject matter and other aspects of the context of inquiry. Instead, I propose to turn to some well-known examples of principles in mathematics.

The natural numbers are often claimed to be especially transparent. One version of such a claim would be the claim that the axioms of arithmetic or other elementary arithmetic statements are self-evident. Let us consider first the elementary Peano axioms: Every natural number has a successor, the successor is never 0, and if $Sx = Sy$, then $x = y$. We might agree that if one doesn't accept these statements, then one doesn't understand the concept of natural number, at least not as we do. But consider someone skeptical about arithmetic, for example someone who holds a version of strict finitism that rejects the infinity of the natural numbers. Such a person will respond to the claim of self-evidence that the understanding of the notion of natural number that we presume is illusory, that in the sense in which we mean it natural numbers do not exist; we do not have the concept of natural number we claim to have. So at least in relation to such skeptical views, the claim that these axioms are self-evident is not very helpful.

Another observation about them is that although the elementary axioms are accepted as axioms of arithmetic, there are some contexts in which one defines the arithmetic terms in them and proves them. This is true of the standard development of arithmetic in axiomatic set theory, and it is also true of the logicist construction of arithmetic. Frege is credited with proving them in second-order logic from his criterion of identity for cardinal numbers, now popularly known as "Hume's principle." Particularly given the power of second-order logic, it would be difficult to maintain that this derivation obtains the elementary Peano axioms from something more evident. Both of these cases are cases of putting the arithmetic axioms into a more general setting, in the one case of course set theory and in the other a theory of cardinality applying to whatever the variables in second-order logic range over.

These somewhat banal observations illustrate a general fact (which applies also to further arithmetic axioms): In mathematical thought and practice, the axioms of arithmetic are embedded in a rather dense network, in which their primitives may mostly play the role of primitives but do not always do so, and likewise the axioms do not always play the role of premises. Moreover, the process by which they have come to be as evident as they are includes also the deduction of consequences from them, consequences which in some cases were at the time when the axioms were formulated found quite as evident as the axioms themselves. We might add some other kinds of considerations, such as the fact that the Dedekind-Peano axioms in the second-order setting characterize a unique structure.

A whole network of relations thus serves to buttress the evident character of the axioms of arithmetic. Some of these relations lead us into applications, and it has been argued that this exposes arithmetic to the possibility of empirical refutation. This is a complex and contested matter, which I will not go into here.

It may be questioned, however, how well the dialectical picture sketched by Goodman for logic and developed by Rawls and others for moral and political theories fits the case of arithmetic. Axiom systems for arithmetic before the work of Dedekind were fragmentary and incomplete, but they did not contain axioms that had to be revised in the light of their consequences. The elementary Peano axioms may not have been recognized in their role as axioms in much earlier times, but surely inferences corresponding to them were made, in particular inferences that would now be rendered as applications of induction or equivalent principles. Some trial and error was no doubt involved in arriving at the right formulations of the induction axiom, but it consisted mostly in coming to recognize induction as a central principle and giving the formulation that characterized the natural numbers. Where there might be uncertainty or controversy about induction is about whether it really needs to be a primitive principle (as in the Poincaré-Russell controversy at the beginning of this century) or what to regard as a well-defined predicate and thus as falling within the scope of the principle. In the latter case, however, we do not find a modification of principles because consequences of them have been found unacceptable but rather questions of a more or less philosophical character, so that where there is some such dialectic is between attitudes toward mathematical axioms and rules and methodological or philosophical principles having to do with constructivity, predicativity, feasibility, and the like.[15]

Arithmetic does illustrate a feature of mathematical principles generally, that there is a decrease of clear and evident character as one introduces more abstract and logically powerful conceptual apparatus. Most of us find it quite

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[15] One might also find a dialectical relation of principles and lower-level mathematical judgments in the earlier history of the analysis of the concept of number, before the time of Dedekind and Frege. I have not gone into this matter and don't feel qualified to have a view of it. The case of Kant, which I have investigated, is one where aspects of his *philosophy* are certainly dated by later developments in the foundations of arithmetic, some basic ideas have value today, but there are not properly arithmetical claims in his writings that have had to be revised, unless one so considers the "metamathematical" claim that arithmetic has no axioms (*Critique of Pure Reason*, A164/B205).

evident that exponentiation is a total function, but is it fully as evident as it is that addition and multiplication are total? Philosophical reasons have been given for casting doubt on exponentiation, and a consistent position that rejects it is possible. If someone maintained that, say, arithmetic with bounded induction but with exponentiation is inconsistent, it is not clear that we could give a definite refutation that would be in no way question-begging.

I conclude that the impression one gets that axioms of arithmetic are evident by themselves, as the application to them of the idea of rational intuition would suggest, is in one way false but in another way true. It is false because the evident character of the axioms is buttressed by the network of connections in which they stand, so that in that respect their evident character does not come just from their intrinsic plausibility. It is true because once formulated clearly, the axioms have not had to be revised because of consequences that mathematicians were not prepared to accept, although when philosophical considerations are brought in we can discern degrees in the evident character of arithmetical principles.

I will be able to discuss only very briefly the case of set theory. It is a remarkable fact that when a set of axioms for set theory was first proposed by Zermelo in 1908, they were questioned in many ways and remained controversial for thirty years or so but emerged from this controversy hardly revised at all: the need for clearer formulations of the axiom of separation arose early, and it was not long after that the axioms were found incomplete in ways that were remedied by introducing the axiom of replacement. The system ZFC as we now know it is essentially in place in Skolem's address of 1922 (Skolem 1923).

Nonetheless the matter of justification of set-theoretic axioms, even those of ZFC before one considers large cardinals or other additions to the axioms, is a complicated one to which I cannot even attempt to do justice here. I shall confine myself to the question how the picture of rational justification developed in §1 applies to them. For someone experienced with set theory, all the axioms of ZFC have what I have called intrinsic plausibility.[16] The axiom of extensionality seems just to mark the theory as being about *sets* as opposed to attributes or other intensional objects; it is as deserving as any of the honorific title "analytic". The axiom of pairing and the existence of a null set (which in some formulations is an axiom) seem as evident as the axioms of arithmetic, and in much the same way. It is tempting to view the axiom of union in that light, but it is not quite so innocent as it seems because of the way it combines with the axiom of replacement to yield large sets.[17] But the cases that are really distinctive of set theory are Infinity, Replacement, Power Set, and Choice. There is a considerable literature on the justification of these axioms, in which a number of considerations play a role: conceptions of what a set is, more global considerations about the universe of sets (the "iterative conception" and ideas about the inexhaustibility of the universe of sets, which allows for extension of any explicitly described proposed universe), analogies of various kinds, methodological maxims, and considerations concerning consequences, that is, the ability of the axioms to yield the established corpus of set theory and its applications in other parts of mathematics as well as the ability to avoid

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[16] This is generally thought to extend naturally to "small" large cardinals, inaccessible and Mahlo cardinals.

[17] Still, someone skeptical of the large sets yielded already in ZF is more likely to question the axiom of replacement than the axiom of union; an example is Boolos 1998.

untoward consequences, in particular paradoxes.

In this case some sort of dialectical relation between axioms and consequences does seem to occur, in spite of the basically unrevised character of Zermelo's axioms. I think we can see such a relation at work in the familiar history of the reception of the axiom of choice, even though its outcome was that the axiom as originally stated by Zermelo became a quite established part of set theory. The fact that it implies that every set can be well-ordered and other existential claims that could not be cashed in by defining the set claimed to exist was regarded by many mathematicians as a reason for rejecting the axiom or at least regarding it with reserve. It did indeed clash with an antecedently reasonable idea of what a set is, roughly the extension of an antecedently meaningful predicate. Perhaps one reason why it was that understanding rather than the axiom of choice that gave way in the end was that the former already did not harmonize with the idea of a set of all sets of natural numbers, which was necessary to the most straightforward set-theoretic construction of analysis.

We may also see ZFC as having won out over various competing proposed frameworks for mathematics, many of them codifiable as subtheories of ZFC. But some that did not have this character, such as ideas for a type-free theory, were never made really workable. However, the jury is probably still out on whether there is a genuinely alternative framework based on category theory. Many people have observed that for most of mathematical practice a much weaker theory than ZFC is sufficient, even without the work that has been done by Harvey Friedman, Stephen Simpson, and others to identify the weakest axiomatic framework sufficient for given classical theorems. But there is no convincing ground on which the part left out should be ruled out of court as genuine mathematics. But this is a case where "intuitions" concerning axioms, what is known about what follows from them (and what does not), and philosophical and methodological "intuitions" would enter into the justification of any view one could reasonably hold.

The phenomenon of decreasing clarity and evidence for more abstract and powerful principles is of course in evidence in set theory; indeed it is there where it first came to the consciousness of researchers in the foundations of mathematics, and where one finds the basis for the most convincing case for its being unavoidable. This is most evident in the case of large cardinals, and especially "large" large cardinals like measurables and larger. But already the axioms of Power Set and Replacement are found insufficiently evident by many[18]; I myself have (in Parsons 1995) defended the view that a posteriori considerations, that is their having the right consequences, are an essential part of their justification. But if one grants some force to the more "intuitive" or direct arguments for them, then we have a case where the plausibility of principles is strengthened by the consequences at lower levels that they yield.

§3. *Intuition and perception*. We have so far not considered the matter of the relation of the sort of rational evidence we have been considering to anything that might be called intuition of *objects*. One way of distinguishing intuition would be to hold that for an attitude, however epistemically significant, to count as intuition it should be significantly analogous to perception. And of course perception is perception of objects. In mathematics, visualization of some kind plays a strong role in making ideas clear, in the formulation of conjectures, and in the search for proofs. It is therefore an important accompaniment to intuition in the sense that has concerned us. But again, it involves intentional attitudes to

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[18] On Replacement see Boolos 1998. Predicativism is a form of skepticism about the axiom of Power Set.

mathematical objects and so is distinct from a propositional attitude such as we have been considering. But a tendency to conflate senses of "intuition" that are of a rather different character is reinforced by another factor, the influence of views of broadly Kantian inspiration.

Gödel famously claimed that "we do have something like a perception of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true" (1964, p. 268). Gödel does not seem to have had in mind some manner in which "something like a perception" of *sets* would underlie the evident character of the axioms; the "objects of set theory" for him include *concepts*, and rational evidence of a proposition goes with clear perception of the concepts in it.[19] My own tendency is to distinguish sharply between the understanding of predicates and apprehension of any objects associated with them; for it to be evident to me that every natural number has a successor is no doubt an exercise of the concept of number, but I would not characterize it as apprehension or "perception" of that concept; it can be just as evident to someone who is skeptical about the whole idea of concepts. We might still pose the question whether there is something that could reasonably be called intuition of objects in the cases of rational evidence in mathematics that we have discussed, and more generally ask what the analogies are with perception.

A factor that makes "rational intuition" *rational* is the absence of an accompanying event like an external perception. In some cases, such as thought-experiments, there may be a more internal event playing the same role; in the cases of Hilbertian intuition discussed in other writings of mine (e. g. those cited in note 8), there is an accompanying exercise of imagination or perception, but

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[19] See Parsons 1995a, where I discuss Gödel's views at length.

even in the latter case the abstract object does not exercise causal influence on the subject, and any intuitive knowledge that results could in principle have been obtained without that particular occasion of perception or any perception of that individual object. Hilbertian intuition or any other intuition having to do with the form of objects of perception is, however, clearly intermediate between perception and more purely rational cases in this respect.

We have thus a definite disanalogy with perception. That is, however, hardly the end of the matter. There is an analogy in that if a proposition is accepted as intrinsically plausible it is, like a perceptual judgment, not accepted as the conclusion of an argument; it stands to a certain degree on its own. And the fact that we stressed above, that such propositions in the end owe some of their evident status to a network of relations in which they stand to other statements, is not so great a disanalogy as one might at first think. Perceptual judgments, as has been stressed by many philosophers, also have to stand in inferential relations to the rest of our knowledge. That is illustrated by the case of judgments about familiar objects before our eyes, where just the fact that we already know a lot about the object adds to the evidence of the judgment.

Another analogy with perception is claimed in some writing on rational intuition by George Bealer.[20] Bealer insists that intuitions are "seemings" and not beliefs; I could have the intuition that *p*, come to believe that '*p*' is false, and yet it would still seem to me that *p*; in that sense I would still have the intuition; to use

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[20] Bealer 1996, p. 268, 1996a, pp. 5-6. The case is of interest because it is not in general part of Bealer's conception of intuition that it should be analogous to perception.

Gareth Evans's term, intuition is "belief-independent."[21] This is just the case with perceptual illusions; in the Müller-Lyer diagram, it *looks* as if the lines are unequal even if I have just measured them and assured myself that they are really equal, so that I do not *believe* that they are unequal. Likewise, according to Bealer, it still seems to him that the universal comprehension axiom of "naive set theory" is true, even though he has not believed it since he first learned of Russell's paradox.

I don't find Bealer's example convincing; it is not clear what definite statement it would be that thus seems to be true; the inconsistency of the schema, or of the specific instance that gives rise to the paradox, is too obvious. But I am inclined to agree that phenomena of this kind do occur in the rational domain. The analogy is just the one that Kant drew in order to explain his conception of "transcendental illusion", which is central to his explanation of the tendency of our minds toward speculative constructions in metaphysics that cannot yield theoretical knowledge.[22] Kant's view exemplifies a constraint that any theory that allows for rational seemings that persist when the illusion has been exposed should conform to: it should be a natural tendency of *reason* that gives rise to the illusion, not a limitation of human beings of some other kind. I think something like this may be at work in intuitions about truth and related notions that generate semantic paradoxes, but then the illusion is of the same general type as what Kant attempted to characterize. That is, they have to do with some kind or other of "absolute" or "unconditioned". In the case of truth, they would concern either an absolute totality of all propositions or a total semantics for the language

[21] Evans 1982, p. 123.

[22] *Critique of Pure Reason*, A297/B353-4.

one is using or might come to use. That is a rather special case, although it is relevant to set theory where there is something like a transcendental Idea of absolute infinity, or of absolutely all sets. In morality, illusions in the sphere of reason are common, but it would be very difficult to argue that there are genuine illusions of reason, since the most likely distorting factors are nonrational: inclinations, disposition toward sin, and social and cultural conditioning. I am skeptical of the prospects of finding cases of illusions of reason of a kind really different from the Kantian; I certainly don't know of any at present.

There is still the question whether there are other respects in which the evident character of the principles we have discussed in arithmetic and set theory has a perceptual character. We have mentioned the role of visualization in mathematical thought, but it is not clear from that role that it has any more fundamental epistemic force. In the case of arithmetic, I have argued elsewhere that it does, on the ground that arithmetic has intuitive models in a sense of "intuitive" with a quite close relation to perception, and that appeal to such a model provides a natural way of convincing oneself that arithmetic with an infinity of natural numbers is not vacuous.[23] I won't repeat any of this here. I will, however, ask briefly whether it can be extended. An intuitive model of the hereditarily finite pure sets could be obtained in much the same way. But in both these cases the objects of intuition are not properly numbers or sets but just representatives of the structure. Still, the idea of intuition of finite sets of manageable size founded on perception of their elements is one that could be defended phenomenologically up to a point; it was already discussed more than

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[23] See the papers cited in note 8, particularly Parsons 1980.

a hundred years ago by Husserl.[24] For the object of one's intuition to be a set and not a sum, or a set of some other elements, would require that the subject bring to the situation a specific conceptualization, but that is already true of Hilbertian intuition. However, one faces considerable difficulties in developing the idea so as to make intuitively evident a theory of hereditarily finite sets. It would take us too far afield to go into this, because the question one would most naturally ask is whether it is possible to go further in making set theory intuitive.

I think it is quite obvious that any conception of intuition of *sets* or, as Hao Wang described it, intuitive overview of a plurality,[25] would have to rest on extension by analogy of ideas from perception of spatio-temporal objects or intuition of objects like Hilbert's strings or finite sets that are of the scale of spatio-temporal objects. Such analogies can be very helpful in helping us to grasp abstract conceptions. But do they really give intuitive evidence to axioms of set theory, or, as Wang attempted, offer a criterion to distinguish pluralities that form sets from others that do not? The idealization required for the conception of intuition involved is a very strong one, just because of the remoteness from spatio-temporal experience of the very large totalities that arise in higher set theory. A mind that had such powers of intuition would have to transcend not just the limitations that we are familiar with in ourselves, but the scale of space-time itself. We can form some sort of abstract and schematic conception of such a mind, and perhaps we do not need to suppose it in some way "metaphysically" possible that *we* should have had such powers, but it is not clear that such a conception adds anything to the more abstract and purely

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[24] Husserl 1891; cf. Husserl 1948, §61.

[25] Wang 1974, chapter vi.

structural conceptions embodied in set theory itself. Our intuitions (in the more purely rational sense that has been our main theme) about the *objects* of such extravagant possibilities of intuition are firmer and more reliable than our intuitions about such intuition itself or about a mind that might have such powers. I won't pursue this matter further because I did so elsewhere some time ago.[26]

The reader might ask how the views presented in this paper differ from Gödel's. The main differences are three: First, I don't think our finding propositions in a field like set theory intrinsically plausible or even evident is strongly analogous to perception, in particular that it necessarily involves a perception-like attitude toward concepts. Second, Gödel seems to regard intuition and confirmation by consequences as quite separate grounds for accepting an axiom, whereas my own view is that they go together and that in set theory they are dialectically related. Third, perhaps most important, Gödel makes stronger claims for rational intuition, giving the impression that it can by itself make set-theoretic axioms evident. Still, there are significant areas of

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[26] Parsons 1977, pp. 275-279. However, I failed at the time to notice the evidence that Gödel endorsed Wang's basic idea; see Wang 1974, p. 189. Gödel discusses such a highly idealized concept of intuition in remarks reported by Wang; see remarks 7.1.17-19 in Wang 1996, p. 220. But I doubt that Wang derived the idea that "multitudes" that can be "overviewed" in an intuitive way are sets from Gödel; he states that chapter vi of Wang 1974 was "written largely during the academic year 1969-70" (p. xiii), and his extended conversations with Gödel took place in 1971-72. But something more definite on this point could be learned from documents.

agreement: That something like what he describes exists and has to be given some epistemic weight, that some popular ways or explaining it away are unsuccessful, that in the case of mathematical principles there is a connection between understanding and plausibility or evidence.

In these remarks Gödel's realism was not mentioned. The significance that perception of concepts has for him undoubtedly goes with a realistic view about concepts as well as mathematical objects. But Gödel makes clear that the application of his conception of intuition is not confined to a realistic understanding of mathematics; in particular, understanding of the concepts of intuitionist mathematics also gives rise to intuition. I think I am entitled to put questions about realism to one side in this discussion, although giving credence to rational intuition does involve a minimal realism: If one's intuitions are to the effect that certain objects exist, then giving credence to them involves affirming that the objects do exist. But that doesn't say just what "existence" amounts to in this context, or what sense is to be given to the idea that the existence of these objects is independent of us.

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